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Rules of Differentiation:

(A) Functions of one variable

(i) Constant function rule:

The derivative of a constant function $Y = f(x) = k$ is identically zero, i.e. is zero for all values of x.

$$\frac{dy}{dx} = 0, \quad \frac{dk}{dx} = 0, \quad f'(x) = 0$$

The graph of a constant function, say, a fixed cost function $C_F = f(Q) = \$1200$ is a horizontal straight line with a zero slope throughout. Correspondingly, the derivative must also be zero for all values of Q.

$$\frac{d}{dQ} C_F = \frac{d}{dQ} 1200 = 0 \quad \text{or} \quad f'(Q) = 0.$$

(ii) Power function rule:

The derivative of a power function $Y = f(x) = x^n$ is nx^{n-1}.

$$\frac{d}{dx} x^n = nx^{n-1}$$

Ex $Y = x^3 \Rightarrow \frac{dy}{dx} = \frac{d}{dx} x^3 = 3x^2$

$Y = x^9 \Rightarrow \frac{d}{dx} x^9 = 9x^8$.

$\Rightarrow$ Power function rule generalized

When a multiplicative constant c appears in the power function, so that $f(x) = cx^n$, its derivative is

$$\frac{d}{dx} cx^n = cnx^{n-1}$$

Ex $f(x) = 4x^3$

$\Rightarrow f'(x) = 12x^3$

Ex $f(x) = 3x^{-2}$

$\Rightarrow f'(x) = -6x^{-3}$

Ex $Y = 2x$

$$\frac{dy}{dx} = 2x^0 = 2.$$

(B) Two or more functions of the same variable

(i) Sum-Difference Rule:

The derivative of sum (difference) of two functions is the sum (difference) of the derivatives of the two functions.

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x) = f'(x) \pm g'(x)$$

Ex $Y = 5x^3 + 9x^3$

$$\frac{dy}{dx} = \frac{d}{dx}(5x^3 + 9x^3) = \frac{d}{dx}5x^3 + \frac{d}{dx}9x^3 = 15x^2 + 27x^2 = 42x^2.$$

Ex $Y = 2x^3 + 13x^3 - x^3$

$$\frac{dy}{dx} = \frac{d}{dx}(2x^3 + 13x^3 - x^3) = 6x^2 + 39x^2 - 3x^2 = 42x^2.$$

(ii) Product Rule:

The derivative of the product of two (differentiable) functions is equal to the first function times the derivative of the second function plus the second function times the derivative of the first function.

$$\frac{d}{dx}[f(x)\,g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

$$= f(x)\,g'(x) + g(x)\,f'(x)$$

Ex $Y = (2x+3)(3x^2)$

$$\frac{d}{dx}[(2x+3)(3x^2)] = (2x+3)(6x) + (3x^2)(2) = 18x^2 + 18x.$$

(iii) Quotient Rule:

The derivative of the quotient of two functions, $f(x)/g(x)$ is

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Ex $$\frac{d}{dx}\left(\frac{2x-3}{x+1}\right) = \frac{2(x+1) - (2x-3)(1)}{(x+1)^2} = \frac{5}{(x+1)^2}$$

(C) Functions of Different Variables

Chain Rule :

If we have a function $z = f(y)$, where y is in turn a function of another variable x, say, $y = g(x)$, then the derivative of z with respect to x is equal to the derivative of z with respect to y, times the derivative of y with respect to x.

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = f'(y)\, g'(x).$$

Ex $z = 3y^2$, where $y = 2x + 5$, then

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = 6y(2) = 12(y) = 12(2x+5)$$

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